Summary of Algebra in
Population Genetics Section

1. Genotype and gene frequencies

For an autosomal locus, A, with two alleles, A_1 and A_2, there are three possible genotypes A_1A_1, A_1A_2, and A_2A_2. Then let

n_1 = number of A_1A_1 genotypes in the population
n_2 = number of A_1A_2 genotypes in the population
n_3 = number of A_2A_2 genotypes in the population

If N is the total number of organisms in the population, the frequencies of the three genotypes are

$$x = \frac{n_1}{N} \quad \text{(frequency of } A_1A_1\text{)}$$

$$y = \frac{n_2}{N} \quad \text{(frequency of } A_1A_2\text{)}$$

$$z = \frac{n_3}{N} \quad \text{(frequency of } A_2A_2\text{)}$$

In a diploid population of size N, there are 2N gametes. The number of A_1 allele gametes is $2n_1 + n_2$ and the number of A_2 allele gametes is $n_2 + 2n_3$. Therefore, the frequencies of the two alleles are

$$p = \frac{2n_1 + n_2}{2N} \quad \text{or } x + \tfrac{1}{2}y \quad \text{(frequency of } A_1 \text{ allele)}$$

$$q = \frac{n_2 + 2n_3}{2N} \quad \text{or } \tfrac{1}{2}y + z \quad \text{(frequency of } A_2 \text{ allele)}$$

2. Hardy-Weinberg equilibrium

Where there is random mating, the mating type frequencies are

		Males		
		A_1A_1 (x)	A_1A_2 (y)	A_2A_2 (z)
Females	A_1A_1 (x)	x^2	xy	xz
	A_1A_2 (y)	xy	y^2	yz
	A_2A_2 (z)	xz	yz	z^2

In each cell are the frequencies of the specific mating types, i.e. frequency of A_2A_2 x A_2A_2 is z^2. If reciprocal mating types are treated equally, A_1A_1♂ x A_1A_2♀ is the same as A_1A_2♂ x A_1A_1♀, then the following mating type frequencies are obtained.

Types of Mating	Frequency of Mating type	Frequencies of offspring A_1A_1	A_1A_2	A_2A_2
A_1A_1 x A_1A_1	x^2	x^2	-	-
A_1A_1 x A_1A_2	2xy	xy	xy	-
A_1A_1 x A_2A_2	2xz	-	2xz	-
A_1A_2 x A_2A_2	y^2	$\frac{1}{4}y^2$	$\frac{1}{2}y^2$	$\frac{1}{4}y^2$
A_1A_2 x A_2A_2	2yz	-	yz	yz
A_2A_2 x A_2A_2	z^2	-	-	z^2
Total	1	$(x + \frac{1}{2}y)^2 = p2$	↓	$(\frac{1}{2}y + z)^2 = q^2$

$$2(x + \tfrac{1}{2}y)(\tfrac{1}{2}y + z) = 2\ pq$$

$$p^2 \ :\ 2\ pq\ :\ q^2$$

Remembering that $p = x + \frac{1}{2}y$ and $q = \frac{1}{2}y + z$ and substituting these values in the frequencies of the various types of offspring, it is found that the <u>genotype frequencies are a function of the gene frequency (p^2 : 2 pq : q^2) after one generation of random mating.</u> These genotypes frequencies are known as the Hardy-Weinberg equilibrium frequencies.

3. Mutation

p = frequency of allele <u>A_1</u>
q = frequency of allele <u>A_2</u>
u = probability of mutation <u>A_1</u> to <u>A_2</u>
v = probability of mutation <u>A_2</u> to <u>A_1</u>

The change in the gene frequency of allele A_2 is
Δq = (frequency of A_1) (probability of mutation A_1 to A_2)
  - (frequency of A_2) (probability of mutation A_2 to A_1)
$\Delta q = pu - qv$
If $pu = qv$, $\Delta q = 0$ This is an equilibrium.

The frequency of A_2 at equilibrium can be calculated

$$0 = pu - qv$$
$$0 = (1 - q)u - qv$$
$$0 = u - qu - qv$$
$$q(u + v) = u$$

4. Genetic Drift

The mean change over a number of populations in gene frequency when only genetic drift is acting is zero. $\delta q = \overline{q_1} - q_0 = 0$ As a result, its effect is best evaluated by looking at its variance $\sigma_{\delta q}^2$ which is

$$\sigma_{\delta q}^2 = \frac{p_0 q_0}{2N}$$

One can see that this is a function of gene frequency and of the number of gametes in the population.

5. Inbreeding

Offspring which are the products of inbreeding may carry two genes at a locus which are identical because they are replicates of a single gene of an ancestor. These genes are said to be identical by descent. The probability of genes being identical by descent is called Wright's coefficient of inbreeding, F. F may also be looked at as a measure of the proportion by which heterozygosity is reduced in the population. Thus,

	A_1A_1	A_1A_2	A_2A_2
frequency	p^2	2pq	q^2
change	+ Fpq	- F2pq	+ Fpq
frequency after inbreeding	P^2 + Fpq	2pq – 2Fpq	q^2 + Fpq

Note that genotype frequencies change but gene frequencies stay the same.
In natural populations with random mating the probability of 2 gametes uniting which are identical by descent is $\frac{1}{2N}$. Thus,

$$\Delta F = \frac{1}{2N}$$

Therefore, it appears that in small populations (N = 20 or less), a significant amount of inbreeding can develop in spite of random mating.

6. Migration

M = proportion of migrants
1-M = proportion of non-migrants
Q = gene frequency of A_2 of migrants
q = gene frequency of A_2 of non-migrants
The gene frequency in the next generation is
q_1 = (proportion of non-migrants) (gene frequency of non-migrants) + (proportion of migrants) (gene frequency of migrants)
$q_1 = (1 – M) q + MQ$